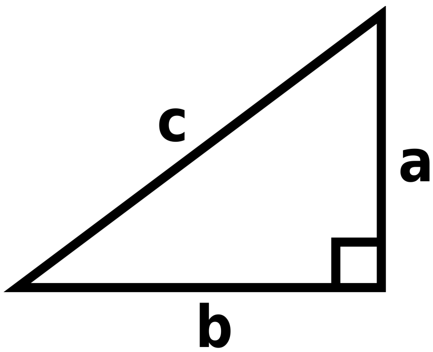
Unit 2: Trigonometry

This lesson is not covered in your workbook. It is a review of trigonometry topics from previous courses.

Pythagorean Theorem

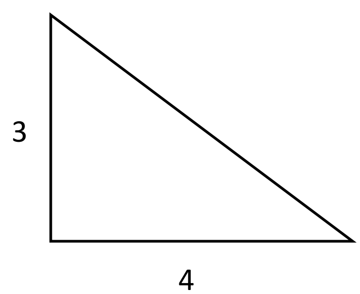
Recall that, for any right angled triangle, the square of the hypotenuse (c) is equal to the sum of the squares of the legs (a and b).



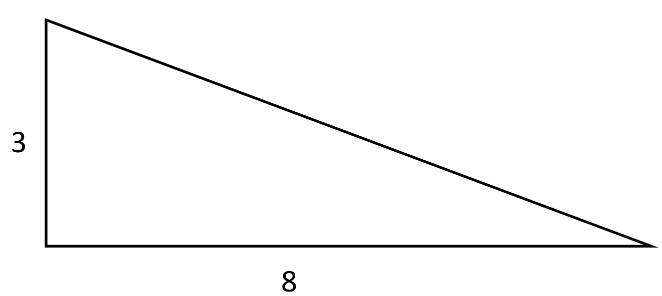
$$c^2 = a^2 + b^2$$

Example 1
For each of the following triangles, find the missing side.

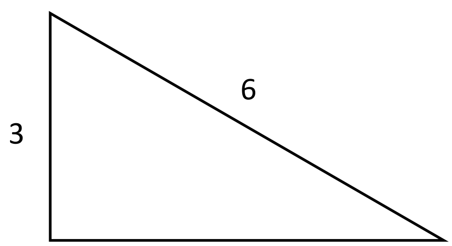
a)



b)



c)



Trigonometric Ratios

Recall the three fundamental trigonometric ratios

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

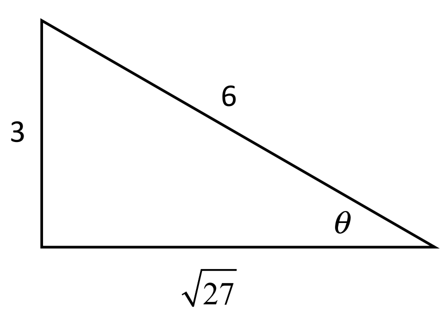
and their inverses

$$\theta = \sin^{-1}\left(\frac{opposite}{hypotenuse}\right) \qquad \theta = \cos^{-1}\left(\frac{adjacent}{hypotenuse}\right) \qquad \theta = \tan^{-1}\left(\frac{opposite}{adjacent}\right)$$

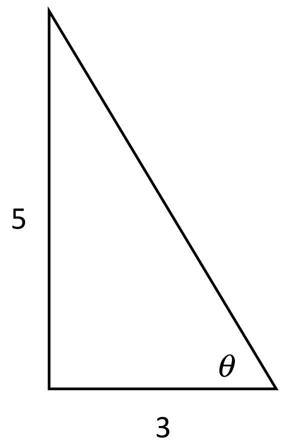
These ratios can be used to solve right angled triangles (i.e. to find missing sides and/or angles).

Example 2
For each triangle below, determine the angle θ.

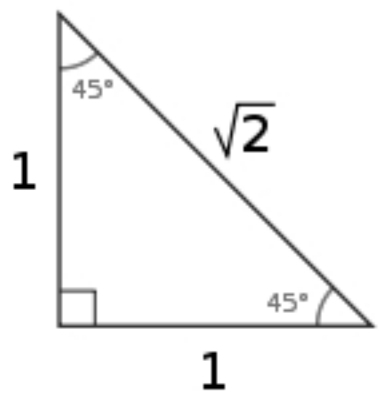
a)

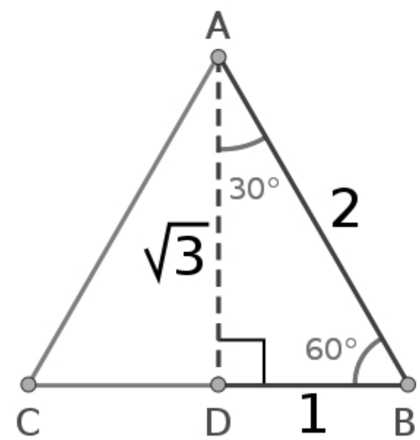


b)



Note: In grade 10, questions like these were done with a calculator. In grade 11 and 12, you are expected to be able to do problems like these without a calculator when they involve certain angles (e.g. $30°$, $45°$, and $60°$). To aid in this, it is useful to memorize the following special triangles.





Solving Trigonometric Equations

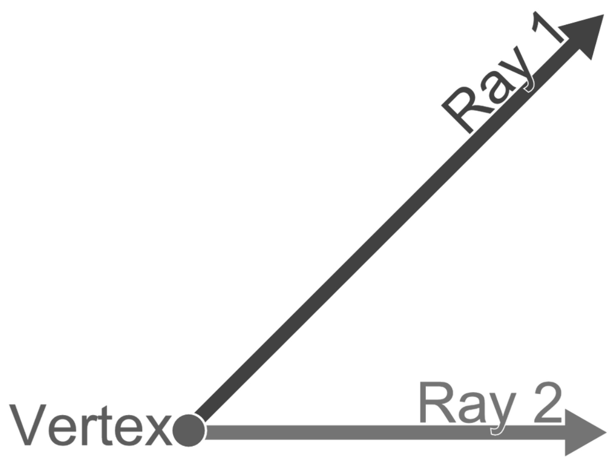
In grade 11, you learned to solve equations involving the trig ratios (i.e. without a diagram).

Example 3
Given that $\cos x = 0.75$, determine the value of x ($0° \le x \le 360°$).

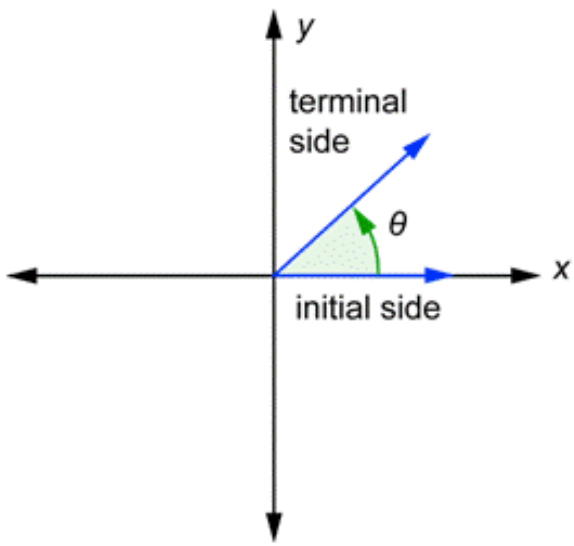
Note: As illustrated by the above example, many trigonometric equations have more than one solution. In order to find all the solutions, we need to review a few additional concepts.

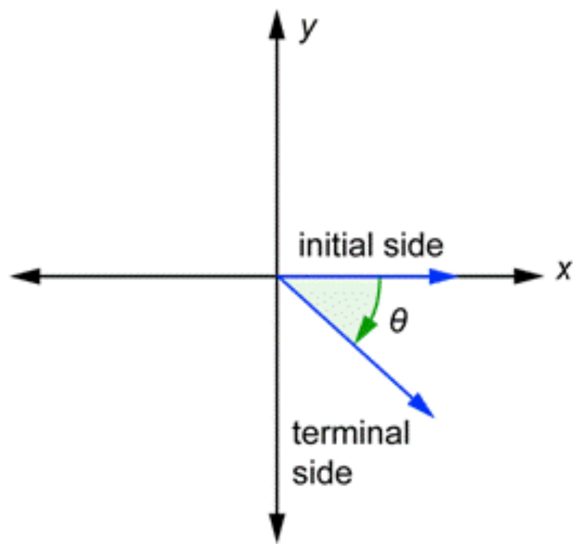
Angles in Standard Position

An angle is the figure formed by two rays (called the sides of the angle) sharing a common endpoint (called the vertex). For example:



An **angle in standard position** is drawn so that the vertex is the origin and one of the rays (called the **initial arm**) is along the positive *x*-axis. The other ray is then called the terminal arm. The angle is measured from the positive x-axis to the **terminal arm**, as shown below.

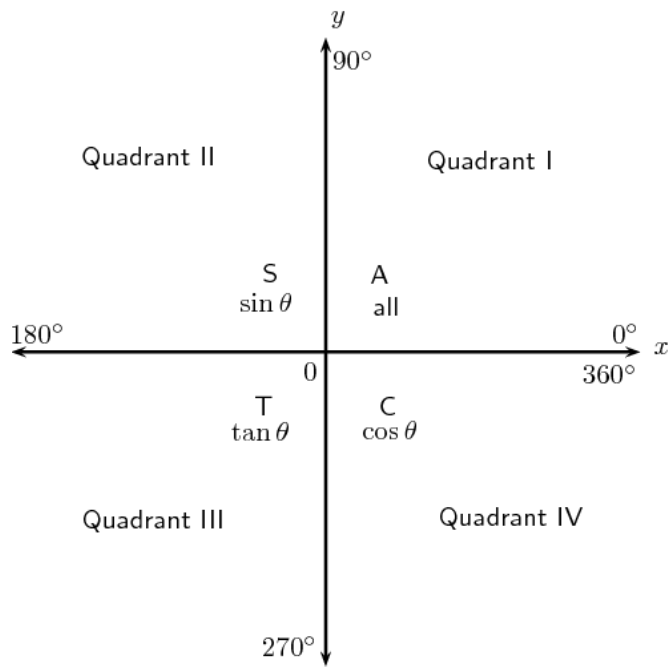




Angles measured counterclockwise are positive, while angles measured clockwise are negative.

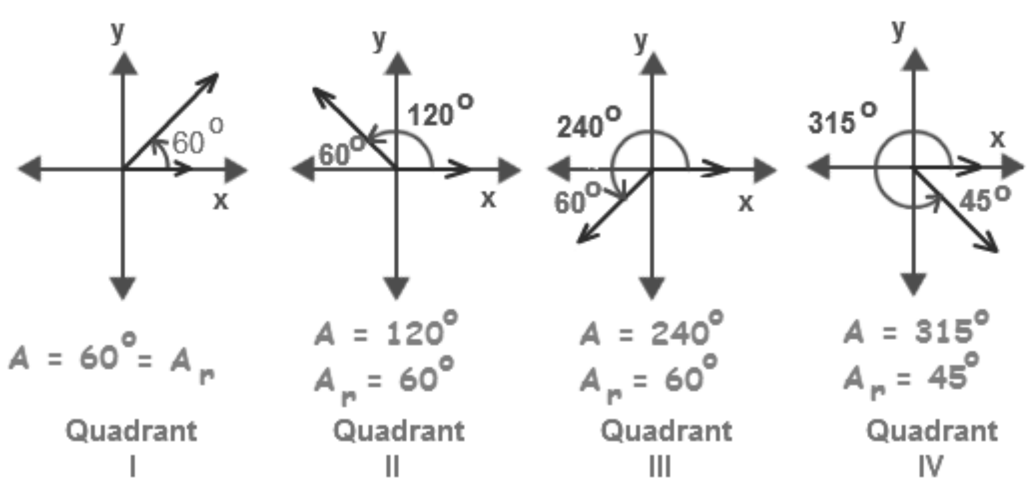
Quadrants

The cartesian coordinate system consists of four quadrants. For any angle in a given quadrant, the trigonometric ratios (sine, cosine, and tangent) will have signs that are predictable. For example, for any angle in Quadrant 2, $\sin\theta$ is positive, while $\cos\theta$ and $\tan\theta$ are negative. This is often referred to as the **CAST rule**, and is illustrated below:



Reference Angle

Associated with every angle drawn in standard position there is another angle called the reference angle. The **reference angle** is the acute angle (less than $90°$) formed by the terminal arm and the x-axis. The image below illustrates this concept:



Note: Angles in Quadrant 1 are their own reference angle.

The concept of a reference angle is useful because an angle and its reference angle have the same trigonometric ratios (though the signs may differ, depending on the quadrant). This can be used to determine the trigonometric ratios of angles outside of Quadrant 1.

Example 4

Determine $\sin\theta$, $\cos\theta$, and $\tan\theta$ for $\theta = 53°$.

Example 5

Determine $\sin\theta$, $\cos\theta$, and $\tan\theta$ for $\theta = 127°$.

Example 6

Determine $\sin\theta$, $\cos\theta$, and $\tan\theta$ for $\theta = 233°$.

Example 7
Determine $\sin\theta$, $\cos\theta$, and $\tan\theta$ for $\theta = 307°$.

Example 8
Solve the equation $\cos x = -0.28$ where $0° \le x \le 360°$.

Homework
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